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Theory of Relation between Hole Concentration and Characteristics of Germanium Point Contacts

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The theory of the relation between the current-voltage characteristic of a metal-point contact to n-type germanium and the concentration of holes in the vicinity of the contact is discussed. It is supposed that the hole concentration has been changed from the value corresponding to thermal equilibrium by hole injection from a neighboring contact (as in the transistor), by absorption of light or by application of a magnetic field (Suhl effect). The method of calculation is based on treating separately the characteristics of the barrier layer of the contact and the flow of holes in the body of the germanium. A linear relation between the low-voltage conductance of the contact and the hole concentration is derived and compared with data of Pearson and Suhl. Under conditions of no current flow the contact floats at a potential which bears a simple relation, previously found empirically, with the conductance. When a large reverse voltage is applied the current flow is linearly related to the hole concentration, as has been shown empirically by Haynes. The intrinsic current multiplication factor, α, of the contact can be derived from a knowledge of this relation.

I. Introduction

IN DISCUSSIONS of the theory of rectification at metal-semiconductor contacts, it is usually assumed that only one type of current carrier is involved: conduction electrons in n-type material or holes in p-type material.[1] In the case of metal-point contacts to high-purity n-type germanium, such as is used in transistors and high-back-voltage varistors, it is necessary to consider flow by both electrons and holes. A large part of the current in the direction of easy flow (metal point positive) consists of holes which flow into the n-type germanium and increase the conductivity of the material in the vicinity of the contact.[2,3] The conductivity is increased not only by the presence of the added holes but also by the additional conduction electrons which flow in to balance the positive space charge of the holes. There is a small concentration of holes normally present in the germanium under equilibrium conditions with no

[1] For a discussion of the nature of current flow in semi-conductors see the "Editorial Note" in *Bell Sys. Tech. Jour.* 28, 335 (1949).
[2] J. Bardeen and W. H. Brattain, *Bell Sys. Tech. Jour.* 28, 239 (1949).
[3] W. Shockley, G. L. Pearson and J. R. Haynes, *Bell Sys. Tech. Jour.* 28, 344 (1949).

current flow. When the contact is biased in the reverse (negative) direction, these holes tend to flow toward the contact and contribute to the current. The hole current is increased if the concentration of holes in the germanium is enhanced by injection from a neighboring contact or by creation of electron-hole pairs by light absorption.

Much has been learned about the effect of an added hole concentration on the current voltage characteristics of contacts from studies with germanium filaments. Part of this work is summarized in a recent article of W. Shockley, G. L. Pearson and J. R. Haynes.[3] These authors have investigated the way the low-voltage conductance of a point contact to a filament of n-type germanium varies with the concentration of holes in the filament and have shown that there is a linear relation between conductance and hole concentration. They have shown that the current to a contact biased with a large voltage in the reverse direction varies linearly with hole concentration. Suhl and Shockley[4] have shown that by applying a large transverse magnetic field along with a large current flow holes may be swept to one side of the filament. Changes in hole concentration produced in this way are detected by measuring changes in the conductance of a point contact.

Shockley[5] has suggested that the floating potential measured by a contact made to a semiconductor in which the concentration of carriers is not in thermal equilibrium may depend on the nature of the contact and differ from the potential in the interior. Pearson[6] has investigated this effect for point contacts on germanium filaments, and has shown that the floating potential is related to the conductance of the contact. This effect provides an explanation for anomalous values of floating potentials measured by Shockley[6] and by W. H. Brattain.[6] They found that potentials measured on a germanium surface in the vicinity of an emitter point biased in the forward direction may be considerably higher than expected from the conductivity of the material.

The purpose of the present paper is to develop the theory of these relations. We are particularly interested in effects produced by changes in hole concentration in n-type germanium resulting from hole injection or photoelectric effects. The equations developed also apply to injected electrons in p-type semiconductors with appropriate changes in signs of carriers and bias voltages. The methods of analysis used are similar to those which have been employed by Brattain and the author in a discussion of the forward current in germanium point contacts[2].

[4] H. Suhl and W. Shockley, *Phys. Rev.* 74, 232 (1948).
[5] W. Shockley, *Bell Sys. Tech. Jour.* 28, 435 (1949), p. 468.
[6] Unpublished.

The problem may be divided into two parts, which can be treated separately:

(a) The first deals with the current-voltage characteristics of the space charge region of the rectifying contact. The current flowing across the contact is expressed as the sum of the current which would flow if the hole concentration in the interior were normal and the current which results from the added hole concentration.

(b) The second is concerned with the current flow in the semiconductor outside the space charge region. In general, both diffusion and conduction

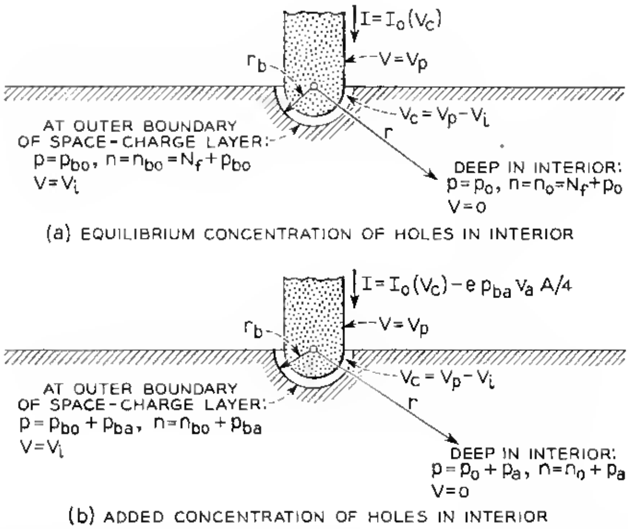

Fig. 1.—Model and notation used for calculation of current flow in low-voltage case.

are important in determining the flow of carriers, although, depending on conditions, one may be much more important than the other. In case the applied voltage and current flow are small, holes in an n-type semiconductor move mainly by diffusion. This situation applies to the problems discussed in the first part of the memorandum. In Section IV we discuss the opposite limiting case of large voltages in which the electron current flowing is so large that the hole current is determined by the electric field and diffusion is unimportant.

The model which is used to investigate the low-voltage case is illustrated in Fig. 1. For purposes of mathematical convenience, the contact is represented as a hemisphere extending into the germanium. Recombination, both at the surface of the semiconductor and in the interior, is

assumed to be negligible so that the lines of current flow are radial. The spherical symmetry of the resulting problem simplifies the matbematics. A calculation is given in an Appendix for a model in which the contact is a circular disk and recombination takes place at the surface. The latter does not give results which are significantly different from tbe simplified model.

Figure 1(a) applies to the case in which the hole concentration deep in the interior has its normal or thermal equilibrium value, p_0. The subscript zero is used to denote values which pertain to this situation. Of a voltage V_P applied to the contact, a part V_c occurs across the space-charge barrier layer of the contact and a part V_i occurs in the body of the semiconductor. Thus V_P represents the voltage of the contact and V_i the voltage in the semiconductor just outside the barrier layer, both measured relative to a point deep in the interior. It should be noted that V_P does *not* include the normal potential drop which occurs across the barrier layer under equilibrium conditions with no voltage applied. In the examples with which we shall deal in the present memorandum, the spreading resistance is small compared with the contact resistance, so that V_i is small compared with V_P. Obviously,

$$V_P = V_c + V_i. \tag{1}$$

When a current is flowing to the contact the hole concentration, p_{b0}, measured just outside of the barrier layer, differs from the concentration deep in the interior, p_0. It is the concentration gradient resulting from the difference between p_{b0} and p_0 which produces a flow of holes from the interior to the contact. In the forward direction, p_{b0} is larger than p_0; in the reverse direction, p_{b0} is less than p_0.

The total current, $I_0(V_c)$, flowing across the contact includes both electron and hole currents. It will not be necessary to distinguisb between these two contributions to the normal current flow across the barrier layer in the subsequent analysis.

Figure 1(b) applies to the case in which the hole concentration deep in the interior has been increased to $p_0 + p_a$ by adding a concentration p_a to the normal concentration, p_0. The concentration just outside the barrier layer is increased to $p_{b0} + p_{ba}$. In addition to the normal current, $I_0(V_c)$, flowing across the contact, there is an additional current of holes resulting from the added hole concentration, p_{ba}, at tbe barrier.

The magnitude of this added hole current is determined in the following way. It is assumed that all holes which enter the barrier region are drawn into the contact by the field existing there. The number of holes

entering the barrier region per second is given by the following expression from kinetic theory:

$$p_b v_a A/4, \tag{2}$$

where v_a is the average thermal velocity, $2(2kT/\pi m)^{1/2}$, of a hole and A is the contact area. This expression gives the average number of particles which cross an area A from one side per second in a gas with concentration p_b. It follows that the current due to the added holes is:

$$I_{pa} = -ep_{ba}v_a A/4. \tag{3}$$

Since, by convention, a current flowing into the semiconductor is positive, a current of holes flowing from the interior to the contact is negative.

The diffusion current resulting from the added holes depends on the difference between p_{ba} and p_a. We shall show in Section III that when p_a is small compared with the normal electron concentration,

$$I_{pa} = 2\pi r_b kT\mu_p(p_{ba} - p_a), \tag{4}$$

where r_b is the radial distance to the outer boundary of the barrier layer and μ_p is the hole mobility. The value of p_{ba} is found by equating (4) and (3), i.e., the added current flowing from the interior to the barrier layer and the current flowing across the barrier layer. This gives

$$p_{ba}/p_a = a/(1 + a), \tag{5}$$

where a, defined by

$$a = 4(kT/er_b)\mu_p/v_a, \tag{6}$$

is the ratio of the velocity acquired by a hole in a field $4kT/er_b$ to thermal velocity. This ratio is generally a small number so that the a in the denominator of (5) can be neglected in comparison with unity. Equation (3) then becomes:

$$I_{pa} = -eap_a v_a A/4 = -p_a kT\mu_p A/r_b. \tag{7}$$

If p_a is not assumed small, a similar procedure may be used but the expressions for I_{pa} in terms of p_a are more complicated than (4) and (7)

It is possible that the added hole current, I_{pa}, will affect the contact in such a way as to change the normal current flowing. If there is such a change, one might expect it to be proportional to I_{pa} as long as I_{pa} is sufficiently small. The total current flow may then be expressed in terms of an "intrinsic α" for the contact as follows:

$$I = I_0(V_c) - \alpha\, I_{pa}(p_a). \tag{8}$$

There is no good theoretical reason to expect that α is different from unity for small current flow in normal contacts unless trapping is important.

Equation (8) is used as the basis for the analysis of the low-voltage data. One important consequence of the equation is that if p_a is different from zero, there is a voltage drop across the barrier layer even though no net current flows to the point. The presence of the added holes in the interior produces a floating potential on the point. The magnitude of this floating potential, V_{cf}, is obtained by setting $I = 0$ in Eq. (8) and finding the value of V_c which solves the equation. This potential can be observed on a voltmeter and is analogous to a photovoltage.

Associated with the floating potential is a change in conductance of the contact. The conductance near $I = 0$, given by

$$G = (dI/dV_c)_{V_c=V_{cf}} = (dI_0/dV_c)_{V_c=V_{cf}}, \tag{9}$$

is just the conductance for normal hole concentration in the interior at an applied voltage equal to V_{cf}. In setting the conductance equal to the derivative of I with respect to V_c, we have neglected the difference, V_i, between V_c, the voltage drop across the barrier, and V_P, the total drop from the contact to the interior. This corresponds to neglecting the spreading resistance in comparison with the barrier resistance.

Equation (8) may be used to relate the floating potential with change of conductance of the contact. The appropriate equations, together with applications to data of Pearson and of Brattain, are given in Section II. In Section III we derive Eq. (4) which relates the added hole current with the added hole concentration in the interior. This relation is used to show that the point conductance G varies linearly with the added hole concentration, p_a. The theoretical expression for conductance is compared with data of Pearson and of Suhl.

In section IV we discuss the dependence of the current-voltage characteristic at large reverse voltages on hole concentration. Under these conditions it is the electric field rather than diffusion which produces the hole current in the body of the germanium. The electron and hole currents are then in the ratio of the electron to hole conductivity. With introduction of an "intrinsic α" for the contact, a simple relation is derived for the dependence of current on hole concentration for fixed voltage on the point. This relation is used to determine α for several point contacts from some data of J. R. Haynes.

II. Floating Potential of Point Contact

In order to get analytic expressions for the floating potential and admittance, it is necessary to make some assumption about the normal cur-

rent-voltage characteristic, $I_0(V_c)$. It is found empirically[7] that as long as V_c is not too large (a few tenths of a volt for a point contact on n-type germanium), it is a good approximation to take:

$$I_0\,(V_c) = I_c\,(\exp(\beta eV_c/kT) - 1), \tag{10}$$

where I_c is a constant for a given contact. Except for the factor β, this is of the form to be expected from the diode theory of rectification. The empirical value of β is usually less than the theoretical value of unity in actual contacts.

If (10) is inserted into (8), the following equation is obtained for the current when there is an added concentration of carriers, p_a, in the interior:

$$I = I_c\,(\exp(\beta eV_c/kT) - 1) - \alpha I_{pa}. \tag{11}$$

Setting $I = 0$ and solving the resulting equation for the floating potential, $V_c = V_{cf}$, we find:

$$V_{cf} = (kT/e\beta)\log\,[1 + \alpha(I_{pa}/I_c)]. \tag{12}$$

The floating potential may be simply related to the conductance corresponding to small current flow. Using Eqs. (9) and (11), we find:

$$G = (dI_0/dV_c)_{V_c=V_{cf}} = (\beta eI_c/kT)\exp\,(\beta eV_{cf}/kT). \tag{13}$$

Since the normal low-voltage conductance is just

$$G_0 = \beta eI_c/kT, \tag{14}$$

we have

$$G = G_0\exp\,(\beta eV_{cf}/kT). \tag{15}$$

By using (12), G can be expressed in terms of p_a. This relation is given and compared with experiment in Section III. Equation (15) may be solved for the floating potential:

$$V_{cf} = (kT/e\beta)\log\,(G/G_0). \tag{16}$$

It should be noted that (16) does not involve p_a directly. Thus it is possible to determine V_{cf} from a measurement of the change in conductance without direct knowledge of the added hole concentration. It holds for large as well as small p_a.

The logarithmic relation (16) between floating potential and conductance has been demonstrated by an experiment of Pearson. The experi-

[7] See H. C. Torrey and C. A. Whitmer, "Crystal Rectifiers", McGraw-Hill Company, New York, N. Y., (1949), p. 372–377.

mental arrangement is illustrated in Fig. 2. Holes are injected into a germanium filament by an emitter point and the circuit is closed by allowing the current to flow to the large electrode at the left end. The right end of the filament is left floating. Some of the injected holes diffuse

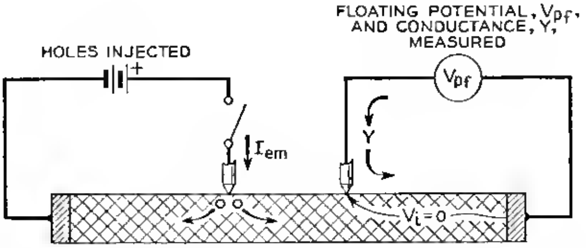

Fig. 2.—Schematic diagram of experiment of G. L. Pearson to investigate relation between floating potential and impedance of point contact.

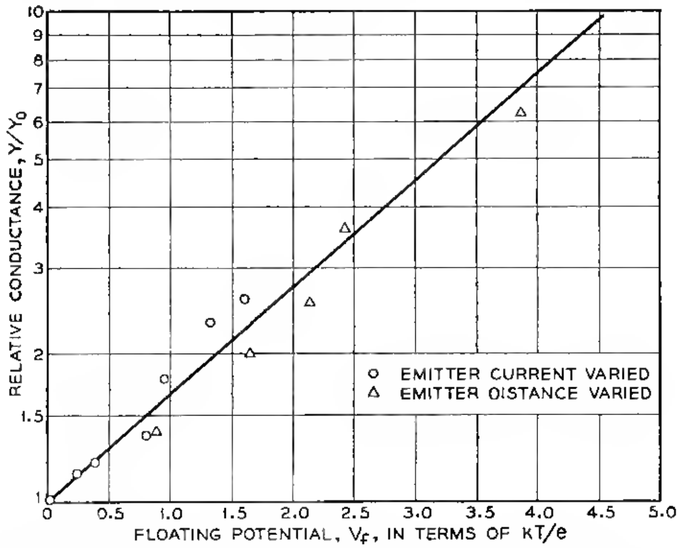

Fig. 3.—The relationship of admittance ratio to potential, measured at a point on a germanium filament into which holes are emitted, with no current flow, from G. L. Pearson's data of September 21, 1948.

down the filament and increase the local concentration in the neighborhood of the probe point. This concentration can be varied by changing the emitter current and also by changing the distance between emitter and probe. Both the floating potential and the conductance between the probe point and the large electrode on the right end were measured. Under the conditions of this experiment, the potential drop in the in-

terior of the floating end of the filament is small. The small drop which does exist results from the difference in mobility between electrons and holes. Almost all of the potential difference between the probe and the right end is the floating potential, V_{cf}, across the barrier layer of the probe point.

Pearson's data are plotted in Fig. 3. The data can be fitted by an equation of the form (16) with $\beta = 0.5$.

The difference in potential between a floating point contact and the interior which exists under non-equilibrium conditions explains anomalously high values of probe potential which were sometimes observed by Shockley and by Brattain in the vicinity of an emitter point operating in the forward direction. As an example of a case in which the effect is

TABLE I

Measurements of probe potential, V_{pf}, at a contact on an etched germanium surface .005 cm from a second contact carrying a current I. The conductance of the probe point is G_P. The voltage drop across the probe contact, $V_{pf} - V_i$, at zero current is calculated from $V_{pf} - V_i = 2.5(kT/e) \log (G_p/G_0)$. Data from W. H. Brattain.

I amps	V_{pf} volts	V_{pf}/I ohms	G_p mhos	G_p/G_0	$\log (G_p/G_0)$	$V_{pf} - V_i = .062 \log (Y_p/Y_0)$	V_i volts	V_i/I ohms
2.0×10^{-3}	0.189	94	8.3×10^{-4}	6.9	1.93	0.120	0.069	35
1.0	0.141	141	5.0	4.2	1.435	0.090	0.051	51
0.5	0.096	190	3.3	2.8	1.030	0.064	0.032	64
0.2	0.052	260	2.2	1.8	0.588	0.037	0.015	75
0.1	0.030	300	1.7	1.4	0.336	0.021	0.009	90
−0.1	−0.0096	96	1.2	1.0				
−0.2	−0.0186	93	1.2	1.0				
−0.5	−0.044	88	1.25	—				
−1.0	−0.10	100	1.35	—				

large, some data of Brattain are given in Table I for the experimental arrangement of Fig. 4. Two point contacts were placed about .005 cm apart on the upper face of a germanium block. The surface was ground and etched in the usual way. A large-area, low-resistance contact was placed on the base. The potential, V_P, of one point, used as a probe, was measured as a function of the current flowing in the second point. In this case, the potential on the probe point is produced in part by the V_{cf} term and in part by a potential, V_i, in the interior which comes from the IR drop of the current flowing from the emitter point to the base electrode. Reasonable values are obtained for V_i from measurements of V_P if a correction for V_{cf} is properly made.

The first column of Table I gives the current and the second column the probe potential, V_P, measured relative to the base. The third column gives values of V_P/I. In the reverse direction (negative currents) V_P/I

is approximately constant at a little less than 100. Values of V_P/I in the forward direction are much larger, starting at 300 for $I = 0.1$ ma and decreasing to 94 at $I = 2$ ma. If anything, one would expect a decrease rather than an increase in V_P/I in the forward direction as injection of holes lowers the resistivity of the germanium in the vicinity of the point. We shall show that V_i/I actually does decrease and that the anomalously high values of V_P/I in the forward direction result from the drop, V_{ef},

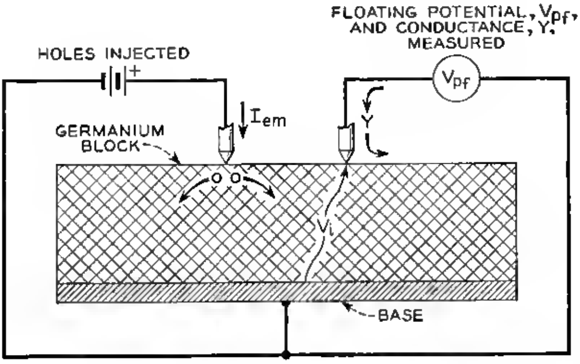

Fig. 4.—Schematic diagram of experiment of W. H. Brattain for measuring floating potential and admittance at point near emitter.

across the barrier layer between the contact point and the body of the germanium. Thus,

$$V_i = V_P - V_{ef}. \tag{17}$$

Values of V_{ef} can be estimated from the change in conductance corresponding to small currents in the probe point. The conductance increases with increasing forward emitter current. Values of V_{ef}, calculated from

$$V_{ef} = 2.5\ (kT/e) \log\ (G_p/G_0), \tag{18}$$

are given in column 6. The value 2.5, chosen empirically to give reasonable values of V_i, is not far from the value 2.0 required to fit Pearson's data in Fig. 1. Values of V_i obtained from Eq. (17) are given in column 7. The ratios V_i/I given in column 8 are reasonable. The decrease in V_i/I with increasing forward current is caused by a decrease in the resistivity of the germanium resulting from hole injection.

In another case, in which no such anomaly was observed in the forward direction, it was found that V_{ef}, calculated from the change in conductance, was small compared with V_p.

There have as yet been no measurements which permit a comparison of the values of β required to correlate probe potential and conductance

with values of β obtained directly from the current-voltage characteristic of the probe. Such a comparison would provide a valuable test of the theory.

III. Low Voltage Conductance of Point Contacts

In this section we calculate the hole current flowing in the body of the germanium from diffusion and find an expression relating change of conductance with added hole concentration. The results shall be applied to data of Pearson and of Suhl. We need to derive Eq. (4) which gives the hole current in terms of the added hole concentrations, p_{ba}, measured just outside the barrier layer, and p_a, measured deep in the interior.

The model which is used for the calculation is illustrated in Fig. 1. The diffusion equation for hole flow is to be solved subject to the boundary conditions that $p = p_b$ just outside the barrier layer and $p = p_i$ at large distances from the contact in the interior. It is assumed that the total current flow is zero or small.

We shall first derive the more general equations[8] which include flow by the electric field as well as by diffusion in order to show the conditions under which the electric field can be neglected. In the body of the semiconductor, conditions of electric neutrality require that the electron concentration, n, be given by:

$$n = N_f + p, \tag{19}$$

where N_f, the net concentration of fixed charge, is the difference between the concentrations of donor and acceptor ions. We shall assume that N_f is constant so that

$$\text{grad } n = \text{grad } p. \tag{20}$$

The general equations for electron and hole current densities, i_n and i_p, are:

$$i_n = \mu_n\,(enF + kT \text{ grad } n) \tag{21}$$

$$i_p = \mu_p\,(epF - kT \text{ grad } p), \tag{22}$$

where F is the electric field strength. By using (19) and (20), and setting $\mu_n = \beta\mu_p$, we can express i_n in the form:

$$i_n = b\mu_p\,(e(N_f + p)\,F + kT \text{ grad } p). \tag{23}$$

The magnitude of F for zero net current,

$$i = i_p + i_n = 0, \tag{24}$$

[8] A discussion of the equations of flow is given in the article by W. van Roosbroeck in this issue of the *Bell System Technical Journal*.

can be obtained by adding (22) and (23) and equating the result to zero. This gives:

$$\frac{eF}{kT} = -\frac{b-1}{N_f b + p(b+1)} \text{ grad } p. \tag{25}$$

The field vanishes for $b = 1$, corresponding to equal mobilities for holes and electrons. For b greater than unity and for equal concentration gradients of holes and electrons, the diffusion current of electrons is larger than that of holes. The field is such as to equate these currents hy increasing the flow of holes and decreasing the flow of electrons.

If (25) is substituted into (22), the following equation is obtained for i_p:

$$i_p = -kT\mu_p \left[\frac{(b-1)p}{N_f b + p(b+1)} + 1\right] \text{grad } p. \tag{26}$$

If recombination is neglected, the hole current is conserved and

$$\text{div } i_p = 0. \tag{27}$$

Using this relation, an equation of the Laplace type can be obtained for p which may be integrated subject to the appropriate boundary conditions. This derivation is given in Appendix B. The results do not differ significantly from those obtained below for p assumed small.

Rather than continue with the general case, we shall at this point assume that $p \ll N_f$ so that the first term in the parenthesis of Eq. (26) is negligible in comparison with unity. This amounts to setting $F = 0$ in Eq. (3) and assuming that the holes move entirely by diffusion. This is a very good approximation in most cases of practical interest and is valid for small i as well as for $i = 0$. We then have

$$i_p = -kT\mu_p \text{ grad } p. \tag{28}$$

The condition div $i_p = 0$ gives Laplace's equation for p:

$$\nabla^2 p = 0. \tag{29}$$

Equation (29) is to be solved subject to the appropriate boundary conditions. For the model illustrated in Fig. 1 we can assume that p depends only on the radial distance r and that

$$p = p_b \text{ at } r = r_b, \tag{30}$$

$$p = p_i \text{ at } r = \infty. \tag{31}$$

The solution of (29) which satisfies (31) is:

$$p = p_i + (I_p/2\pi kT\mu_p r), \tag{32}$$

in which I_p is the total hole current. The boundary condition (30) gives the relation between I_p and p_b:

$$p_b = p_i + (I_p/2\pi kT\mu_p r_b). \tag{33}$$

Since the equations are linear, an equation of the form (33) applies to the hole current due to the added holes as well as to the entire hole current. For the former we have:

$$p_{ba} = p_a + (I_{pa}/2\pi kT\mu_p r_b), \tag{34}$$

which is equivalent to Eq. (4).

In the derivation of Eq. (34) we have neglected recombination at the surface as well as in the interior. In the Appendix we give a solution for a contact in the form of a circular disk and assume that recombination takes place at the surface. The hole concentration then satisfies Laplace's equation subject to more complicated boundary conditions at the surface. The results are not significantly different from those of the simplified model.[9]

Equation (34), or rather its equivalent, Eq. (4), was used in the derivation of Eq. (12) for the floating potential, V_{cf}. If this value for V_{cf} is inserted into Eq. (15), an equation relating the conductance directly with the added hole concentration is obtained:

$$G = G_0 + (\alpha e^2 a v_a A\beta p_a/4kT). \tag{35}$$

This expression may be simplified by substituting for a from Eq. (6):

$$G = G_0 + \alpha\beta\mu_p e A p_a/r_b. \tag{36}$$

By using the expression for the normal conductivity:

$$\sigma_0 = b\mu_p e n_0, \tag{37}$$

the conductance can be given in the form:

$$G = G_0 + (\alpha\beta\sigma_0 A/br_b)(p_a/n_0). \tag{38}$$

If σ_0 is in practical units (mhos/cm), G is in mhos.

We shall compare (38), which gives a linear variation between G and p_a, with experimental data of Pearson[10] and of Suhl. The arrangement used

[9] In the applications, these equations are applied to situations in which the contact is on a germanium filament and there is a flow of current along the length of the filament in addition to the flow to the contact. A question may arise as to whether it is justified to neglect the filament current when discussing flow to the contact. There is no difficulty as long as p_a/n_0 is small compared with unity because the equations are then linear and the solution giving the flow to the contact can be superimposed on the solution giving the flow along the length of the filament. The neglect of the filament current cannot be rigorously justified in case p_a/n_0 is large, as is assumed in the calculations of Appendix B. It is not believed, however, that the exact treatment would yield results which are significantly different.

[10] See reference 3, p. 356 and Fig. 6.

by Pearson is shown in Fig. 5. Two probe points were placed about .009 cm apart near one end of a germanium filament. The concentration

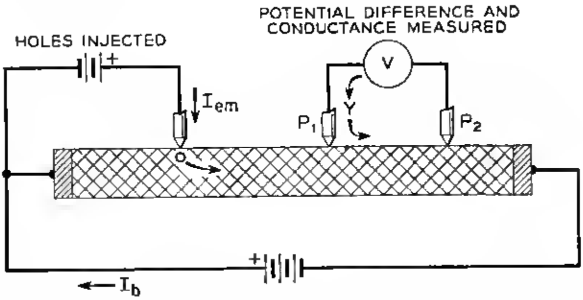

Fig. 5.—Experimental arrangement used by G. L. Pearson to investigate relation between admittance and hole concentration in germanium filament.

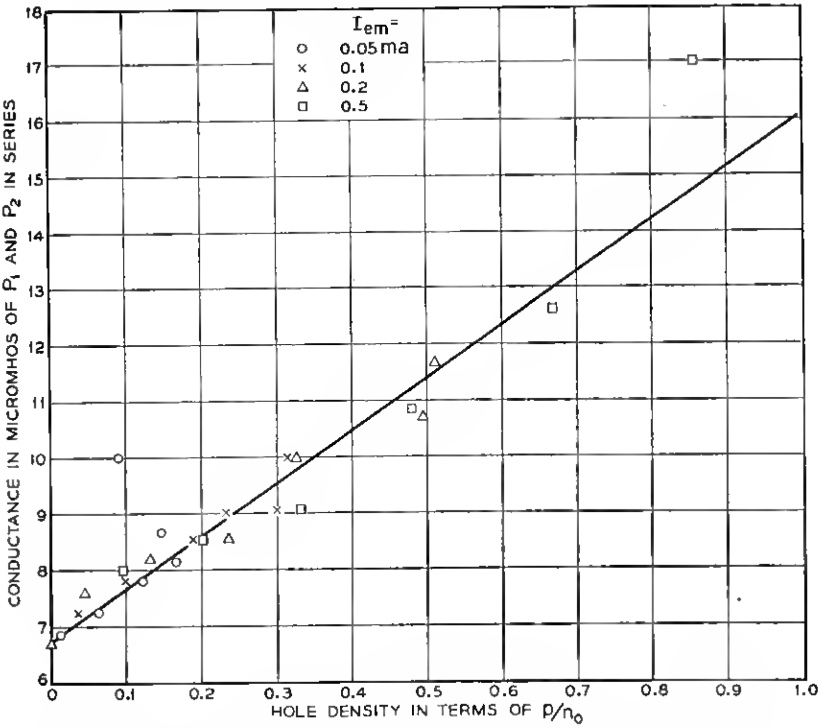

Fig. 6.—The relationship between point admittance and relative hole concentration, for a germanium filament from G. L. Pearson's data of September 28, 1948.

of holes was varied by current from an emitter point near the opposite end of the filament. There was an additional current flowing between

electrodes at the two ends so that the field pulling the holes along the filament could be varied. The concentration of holes was determined from the change in resistivity of that segment of the filament between the two probes. Measurements of admittance were made by passing a small current between the two probes connected in series. The area of the filament is about 1.6×10^{-4} cm² and the normal resistance between the probes about 1800 ohms. The normal conductivity is thus

$$\sigma_0 = .009/(1800 \times 1.6 \times 10^{-4}) = 0.03 \text{ (ohm cm)}^{-1}. \tag{39}$$

As shown in Fig. 6, Pearson finds a linear relation between G and p_a. The line best fitting Pearson's data is

$$G = G_0 + (8 \times 10^{-6})\,(p_a/n_0) \text{ (mhos)}. \tag{40}$$

The theoretical value of the coefficient may be obtained from Eq. (38). Taking

$$\begin{aligned} \alpha &= 1, \quad \beta = 0.5, \qquad \sigma_0 = 0.03 \\ b &= 2.0, \; A = 10^{-6}\text{ cm}^2, r = 5 \times 10^{-4}\text{ cm}, \end{aligned} \tag{41}$$

we get

$$\alpha\beta\sigma_0 A/b\, r_b = 15 \times 10^{-6}\text{ mhos}. \tag{42}$$

Pearson's data, represented by (40), apply to the conductance of two point contacts in series, and the conductance of each one may be about twice that given by (40). Thus the theoretical value is in good agreement with the observed. There is no indication that α differs from unity at low voltage.

Suhl varied the concentration of holes in the vicinity of probe points by application of a transverse magnetic field as well as by injection from an emitter point. The experiment is illustrated in Fig. 7. He used a filament with a cross-section of about $.025 \times .025$ cm. Four probe points were placed along the length of the filament at intervals of about .04 cm. A total current of 4 ma flowed in the filament.

In one experiment, none of this current was injected, so that the concentration of holes was normal in the absence of the magnetic field. Measurements were made of the floating potentials and of the conductances of the probe points. Then a transverse magnetic field was applied and the conductances measured again. We are interested here only in the case of a large field (30,000 gauss) in such a direction as to sweep the holes to the opposite side of the filament. Suhl believes that under these conditions the concentration of holes near the probe points is practically zero. The difference between the conductances with and without the field

then gives the contribution to the conductance from the normal concentration of holes.

In a second experiment 1 ma of the current of 4 ma flowing in the filament was injected from an emitter point near one end of the filament. From the probe potentials, estimates have been made of the change in

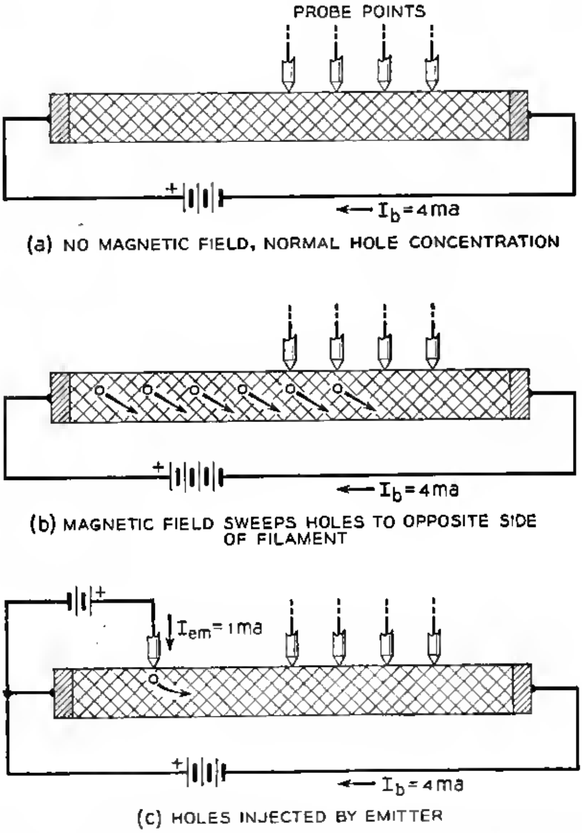

Fig. 7.—Schematic diagram of experiment of H. Suhl to investigate relation between hole concentration and impedance of point contacts.

resistivity and thus of the added hole concentration at the different probe points. Changes in hole concentration from injection have been correlated with changes in admittance of the probe points.

The filament with dimensions .025 × .025 × 0.4 cm has a resistance of 4,600 ohms. The normal resistivity, ρ_0, is then about 7.2 ohm cm. Since the concentration of electrons corresponding to 1.0 ohm cm is about

1.8×10^{15}, the concentration corresponding to a resistivity of 7.2 ohm cm is[11]:

$$n_0 = 1.8 \times 10^{15}/7.2 = 2.5 \times 10^{14}/\text{cm}^3. \tag{43}$$

The product of the equilibrium concentrations of electrons and holes is about 4×10^{26} in germanium at room temperature[12]. Thus, for this sample,

$$p_0 = 4 \times 10^{26}/2.5 \times 10^{14} = 1.5 \times 10^{12}/\text{cm}^3. \tag{44}$$

If there is an added concentration of holes, p_a, resulting from injection, the added conductivity is:

$$\sigma_a = (1 + b)\, e\mu_h p_a = 8.4 \times 10^{-16}\, p_a\,. \tag{45}$$

The resistivity is changed to:

$$\rho = \rho_0\sigma_0/(\sigma_a + \sigma_0) \simeq \rho_0\,(1 - \sigma_a\rho_0), \tag{46}$$

the approximate expression holding if the relative change is small. The resistance per unit length of filament is:

$$R = 1.15 \times 10^4\,(1 - \sigma_a\rho_0). \tag{47}$$

The change in voltage gradient, $dV/dx = RI$, resulting from hole injection is, for a current of 4×10^{-3} amps,

$$\Delta(dV/dx) = d(\Delta V)/dx = -46\rho_0\sigma_a\,. \tag{48}$$

Suhl measured the change in probe potential, ΔV, which resulted when 1 ma of the total current of 4 ma was injected from the emitter instead of having the entire 4 ma flowing between the ends of the filament. His values of ΔV for the four probe points are given in Table II. We have made a plot of these as a function of position and have estimated the gradients at each of the four probe positions. Using these values we have calculated σ_a from Eq. (48) and the corresponding injected hole concentration from Eq. (45). These are given in the last column of the table.

Suhl's measurements of conductances, G, of the probe points are given in Table III. Also given are differences, ΔG, from the normal values with no magnetic field and no injection and also these differences multiplied by n_0/p_a. Values of p_a for the case of hole injection were obtained from Table II. Values of $\Delta G(n_0/p_a)$ are to be compared with the theoretical value,

$$\Delta G(n_0/p_a) = \alpha\beta\sigma_0 A/cr_b\,, \tag{49}$$

[11] These values are based on taking $\mu_n = 3500$ cm²/volt sec and $\mu_p = 1700$ cm²/volt sec, as measured by J. R. Haynes. They correspond to room temperature (295°K).

[12] This value is obtained from an intrinsic resistivity of about 60 ohm cm for *Ge* at room temperature and the mobility values in reference 11.

from Eq. (38). Taking $\alpha = 1$, $\beta = 0.5$, $\sigma_0 = 0.14$, $b = 1.5$, $A = 10^{-6}$ and $r_b = 5 \times 10^{-4}$, we get

$$G(n_0/p_a) \sim 100 \text{ micromhos.} \tag{50}$$

This value is of the same order as the values obtained from Suhl's data listed in Table III. There is a large scatter in the latter and the values are not consistent. It has been suggested that the abnormal values may result from local sources of holes.

Table II

Calculation of hole concentrations from probe potential measurements. ΔV measures potential difference resulting from hole injection of 1 ma when total current is kept at 4 ma; data from H. Suhl.

Point No.	Relative Position (cm)	ΔV (volts)	$\frac{d\Delta V}{dx}$ (volts/cm)	$\rho_0\sigma_a$	σ_a (mhos)	p_a (cm^{-3})
#6	0	−.04	−0.6	.013	.0018	2.2×10^{12}
#5	.044	−.073	−1.10	.024	.0033	4.0
#4	.084	−.13	−1.8	.039	.0054	6.5
#3	.12	−.21	−2.5	.055	.0077	9.0

Table III

Changes in conductance resulting from application of magnetic field and from hole injection. Units are micromhos. Data from H. Suhl.

Point	No Field	With—30,000 gauss field			With hole injection		
	G	G	ΔG	$\Delta G \frac{n_0}{(-p_0)}$	G	ΔG	$\Delta G \frac{n_0}{p_a}$
#6	17.2	16.4	−0.8	130	22.5	7.8	880
#5	6.55	4.35	−2.2	365	7.0	0.45	28
#4	3.7	3.2	−0.5	80	5.1	1.4	54
#3	13.0	9.2	−3.8	630	19	6	165

IV. Hole Flow for a Collector with Large Reverse Voltage

Haynes has shown that there is a linear relation between the current to a collector point operated in the reverse direction and the concentration of holes in the interior of a germanium filament. Under the conditions of his experiment, the current flowing to the collector point is small compared with the total current flowing down the filament, so that the collector current does not alter the concentrations very much. Holes are injected into the filament by an emitter point placed near one end, and the concentration is determined from the change in resistance of the filament in the neighborhood of the collector point.

Haynes' measurements may be fitted by an empirical equation of the following form:

$$I = I_0[1 + \gamma p_a/n_0)], \tag{51}$$

in which I_0 is the normal collector current flow for a given collector voltage, I is the collector current flowing for the same collector voltage when the hole concentration is increased by p_a, and n_0 is the normal electron concentration. Values of I_0 and γ for four different formed phosphor-bronze collector points are given in Table IV. The collector bias is -20 volts in each case. It can be seen that the variations in γ are much less than those in I_0. It will be shown below that γ is related to the intrinsic α of the point contact.

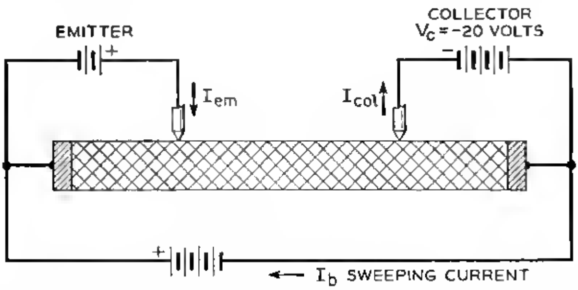

Fig. 8.—Experimental arrangement used by J. R. Haynes to determine relation between hole concentration and current to collector point biased with large voltage in reverse direction.

In Haynes' experiment, holes are attracted to the collector by the field produced by the electron current and diffusion plays a minor role. In contrast to the preceding examples, the terms involving the field F in Eqs. (21) and (22) are large and the diffusion terms represented by the concentration gradients are small. It follows from (21) and (22) that the ratio of electron to hole current density is then:

$$i_n/i_p = bn/p, \tag{52}$$

which is equal to the ratio of the electron and hole contributions to the conductivity. If n and p do not vary with position, the ratio is the same everywhere and equal to the ratio of total electron and hole currents, I_n and I_p:

$$I_n/I_p = i_n/i_p = bn/p. \tag{53}$$

The currents I_n and I_p can also be related to the intrinsic α for the contact by use of an equation of the form:

$$I = I_{n0} + \alpha I_p, \tag{54}$$

in which I_{n0} is the electron current for zero hole current. The electron current is:

$$I_n = I_{n0} + (\alpha - 1)I_p. \tag{55}$$

Thus we have

$$\frac{I_n}{I_p} = \frac{I_{n0} + (\alpha - 1)I_p}{I_p} = \frac{bn}{p} = \frac{b(N_f + p)}{p}. \tag{56}$$

This equation may be solved for I_p to give:

$$I_p = pI_{n0}/(bN_f + (\alpha - 1 - b)p). \tag{57}$$

The term $(\alpha - 1 - b)p$ is generally small compared with bN_f and may be neglected. We thus have approximately for p/N_f small and $N_f \simeq n_0$,

$$I = I_{n0} + \alpha I_p = I_{n0}[1 + (\alpha p/bn_0)]. \tag{58}$$

When expressed in terms of the normal current,

$$I_0 = I_{n0}[1 + (\alpha p_0/bn_0)], \tag{59}$$

the equation for I is of the form (51):

$$I = I_0 [1 + (\alpha p_a/bn_0)]. \tag{60}$$

From a comparison of (51) and (60) it can be seen that:

$$\gamma = \alpha/b \text{ or } \alpha = b\gamma. \tag{61}$$

Values of α determined from empirical values of γ for the four point contacts of Haynes are given in Table IV. The values are of a reasonable order of magnitude for formed collector points.

An estimate of the importance of diffusion can be obtained by comparing the hole current in Haynes' experiments with the hole current which would exist if the electron current were zero, so that holes move by diffusion alone. Equations (28) to (33) apply to the latter case. In addition to (33) we need an equation which expresses the hole current flowing into the contact in terms of the hole concentration, p_b, at the contact. If the reverse bias is large, no holes will flow out and the entire hole current is that from semiconductor to metal as given by an equation similar to (3):

$$I_p = -ep_b v_a A/4. \tag{62}$$

Substituting this value for I_p into equation (33) we get an equation which may be solved for p_b, to give:

$$p_b = ap_i(1 + a) \simeq ap_i, \tag{63}$$

witb a given by Eq. (6). Using (63) for p_b, we get:

$$I_p = kT\mu_h p_i A/r_b = (kT\sigma_0 A/ebr_b)(p_i/n_0). \quad (64)$$

Witb $kT/e = .025$ volts, $\sigma_0 = bn_0e\mu_h = 0.2$ (ohm cm)$^{-1}$, $A = 10^{-6}$ cm^2 and $r_b = 5 \times 10^{-4}$ cm, we get for the diffusion current:

$$I_p = (5 \times 10^{-6})(p_i/n_0) \text{ amps.} \quad (65)$$

Comparing (65) with (57) we see that diffusion of holes will not be important if

$$I_{n0} \gg 5 \times 10^{-6} \text{ amps.} \quad (66)$$

This condition is satisfied in Haynes' experiments.

In the case of point contacts formed to have a high reverse resistance as diodes, I_0 may be of the order of 10^{-7} to 10^{-6} amps at room temperature. Diffusion of holes will then play a role, and the hole current will

TABLE IV

Relation between hole concentration and collector current from data of J. R. Haynes. Data represented by

$$I = I_0(1 + (\gamma p_a/n_0))$$

where I is current flowing to collector point biased at -20 volts and p_a/n_0 is ratio of added hole concentration to the normal electron concentration.

Probe Point	I_0	$a = 2.1\gamma$
0	0.94	4.6
2	0.33	4.4
3	0.54	6.9
4	1.20	4.6

be larger than indicated by Eq. (53). As discussed in reference (4) there is still a question as to the importance of holes in the saturation current observed by Benzer in diodes with high reverse resistance. Experiments similar to those of Haynes would be valuable to determine the influence of hole concentration on reverse current.

ACKNOWLEDGMENT

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APPENDIX A

DIFFUSION OF HOLES WITH SURFACE RECOMBINATION

In the calculation of the diffusion of holes given in Section III of the text it was assumed that no recombination of electrons and boles oc-

curred. In the present calculation it is assumed that recombination occurs at the surface, but not in the volume. This is a good approximation for a point contact on germanium. It is further assumed that the hole concentration is sufficiently small so that Laplace's equation (29) may be used.

The model which we shall use is illustrated in Fig. 9. The contact is in the form of a circular disk of radius ρ on the surface of the semiconductor. Cylindrical coordinates, r, θ, z, are used, with the origin at the center of the disk and the positive direction of the z-axis running into the semiconductor. We calculate the flow due to the added holes, and shall use the symbol p without subscript to denote the added hole concentration.

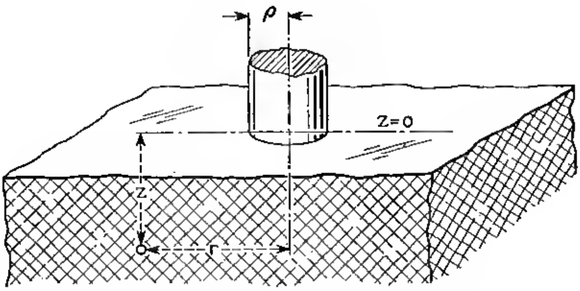

Fig. 9.—Coordinates used for calculation of hole flow to contact area in form of circular disk.

With recombination at the surface, it is necessary to have a gradient in the interior which brings the holes to the surface.

It is assumed that the rate of recombination at the surface is:

$$sp = \text{holes/cm}^2, \tag{1A}$$

where the factor s has the dimensions of a velocity and p is evaluated at the surface $z = 0$. According to measurements of Suhl and Shockley, s is about 1500 cm/sec for a germanium surface treated with the ordinary etch. The current flowing to the surface is:

$$(\mu_p kT/e)(\partial p/\partial z)_{z=0} \text{ holes/cm}^2. \tag{2A}$$

The boundary condition for p at the surface $z = 0$ outside of the contact area is obtained by equating (1A) and (2A). This gives:

$$\partial p/\partial z = \lambda p \text{ at } z = 0,\ r > \rho \tag{3A}$$

where

$$\lambda = se/\mu_p kT, \tag{4A}$$

has the dimensions of a length. For $s = 1500$ cm/sec and $\mu_p = 1700$ cm²/volt sec, corresponding to germanium at room temperature, λ is about 35 cm^{-1}.

The boundary condition on the disk is similar to (3A) except that s is replaced by $v_a/4$ (cf. Eq. (3)). Thus for $r < \rho$,

$$\partial p/\partial z = \lambda_c p \qquad z = 0, r < \rho, \tag{5A}$$

where

$$\lambda_e = v_a e/4\mu_p kT. \tag{6A}$$

Evaluated for germanium at room temperature, λ_e is about 6×10^4.

In order to have a dependent variable which vanishes at infinity, we replace p by:

$$y = p_a - p + \lambda p_a z, \tag{7A}$$

so that $p \to p_a$ for $z = 0$ as $r \to \infty$. The variable y satisfies Laplace's equation subject to the boundary conditions:

$$\partial y/\partial z = \lambda y \qquad z = 0, r > \rho \tag{8A}$$

$$\partial y/\partial z = \lambda_e (y - p_a) \; z = 0, r < \rho \tag{9A}$$

$$y = 0 \qquad r, z \to \infty. \tag{10A}$$

An exact solution of the problem is difficult. We shall obtain an approximate solution which satisfies (8A) but not (9A) and which applies when

$$\lambda\rho \ll 1 \ll \lambda_e\rho. \tag{11A}$$

This approximation is valid for a germanium point contact, since, for $\rho \sim 10^{-3}$ cm,

$$\lambda\rho \sim .035, \lambda_e\rho \sim 60. \tag{12A}$$

We shall first discuss the limiting case for which $\lambda \to 0$ and $\lambda_e \to \infty$. The former implies neglect of surface recombination and the latter

$$y = p_a \text{ for } z = 0, r < \rho. \tag{13A}$$

The problem is the same as that of finding the potential due to a conducting circular disk. The solution of this problem, which is well known, is:

$$y = (2p_a/\pi) \int_0^\infty e^{-zt} J_0(rt) \frac{\sin \rho t}{t} \, dt. \tag{14A}$$

The current flowing to the disk is obtained from integrating:

$$i_z = kT\mu_p(\partial y/\partial z), \tag{15A}$$

over the area of the disk. This gives:

$$I_{pa} = -4\rho p_a kT\mu_p. \tag{16A}$$

The analogous expression for a hemispherical contact area of radius r_b, obtained from (7), is:

$$I_{pa} = -2\pi r_b p_a kT\mu_p. \tag{17A}$$

If a comparison is made on the basis of equal radii, (17A) is larger than (16A) by a factor of $\pi/2$. On the more reasonable basis of equal contact areas, (16A) is larger than (17A) by a factor of $4/\pi$.

An approximate solution which includes surface recombination can be obtained as follows. A solution of Laplace's equation whicb satisfies (8A) and (10A) is:

$$y = \frac{2y_0}{\pi} \int_0^\infty e^{-zt} J_0(rt) \frac{\sin \rho t}{t + \lambda}\, dt. \tag{18A}$$

That (18A) satisfies (8A) may be verified by direct substitution:

$$\left[-\frac{\partial y}{\partial z} + \lambda y\right]_{z=0} = \frac{2y_0}{\pi} \int_0^\infty J_0(rt) \sin \rho t \, dt = 0 \quad \text{for} \quad r > \rho. \tag{19A}$$

$$= (2y_0/\pi)(\rho^2 - r^2)^{-1/2} \quad \text{for} \quad r < \rho. \tag{20A}$$

Expression (18A) satisfies (9A) approximately if λ_c is large. Using (20A) and neglecting λ in comparison with λ_c, we have:

$$y = p_a - (2y_0/\pi\lambda_c)(\rho^2 - r^2)^{-1/2} \quad \text{for} \quad z = 0, r < \rho. \tag{21A}$$

Except for r almost equal to ρ, the second term on the rigbt of (21A) is very small. It is not possible to obtain an explicit expression for y for $r < \rho$. For $z = 0$, $r = \rho$,

$$y = \frac{2y_0}{\pi} \int_0^\infty \frac{J_0(\rho t) \sin \rho t}{t + \lambda}\, dt = y_0 F(\lambda\rho). \tag{22A}$$

The integral, $F(\lambda\rho)$, can be evaluated from a more general integral in Watson's Bessel Functions, p. 433. We have:

$$F(k) = \frac{2}{\pi} \int_0^\infty \frac{J_0(x) \sin x \, dx}{x + k} = \cos k\, J_0(k) + \sin k\, Y_0(k). \tag{22B}$$

The factor multiplying y_0 is unity for $\lambda\rho = 0$, and decreases as $\lambda\rho$ increases. Since y is approximately equal to p_a, we bave, approximately,

$$y_0 = p_a/F(\lambda\rho). \tag{23A}$$

The value of y can also be found for $r = 0$. For $z = 0$, $r = 0$, we have:

$$y = \frac{2y_0}{\pi} \int \frac{\sin \rho t \, dt}{t + \lambda} = \frac{2y_0}{\pi} G(\lambda\rho). \tag{24A}$$

The integral can be expressed in terms of integral sine and cosine functions:

$$G(k) = \frac{2}{\pi} \int_0^\infty \frac{\sin x \, dx}{x + \lambda} = \frac{2}{\pi} \left[-\cos k \left(\mathrm{Si}\, k - \frac{\pi}{2} \right) + \sin k \, \mathrm{Ci}\, k \right]. \tag{25A}$$

If k is not too large, $G(k)$ is nearly equal to $F(k)$, so that y is approximately constant over the area of the disk.

The total current flowing from the contact is found from integrating $kT\mu_p \, (\partial y/\partial z)$ over the disk:

$$I_{pa} = -kT\mu_p y_0 \int_0^\rho \int_0^\infty \frac{4\pi t J_0(rt) \sin \rho t}{t + \lambda} \, dt \, dr \tag{26A}$$

$$= -4kT\mu_p y_0 \int_0^\infty \frac{\rho J_1(\rho t) \sin \rho t}{t + \lambda} \, dt. \tag{27A}$$

The integral can be evaluated with use of the general integral of Watson, to give:

$$I_{pa} = -4\rho kT\mu_p y_0 \, H(\lambda\rho), \tag{28A}$$

where

$$H(k) = \int_0^\infty \frac{J_1(x) \sin x \, dx}{x + k} = -\frac{\pi}{2} [\cos k \, J_1(k) + \sin k \, Y_1(k)]. \tag{29A}$$

Using (23A) for y_0, we have:

$$I_{pa} = -4\rho kT\mu_p p_a [H(\lambda\rho)/F(\lambda\rho)]. \tag{30A}$$

Except for the factor $H(\lambda\rho)/F(\lambda\rho)$, this expression for the current is identical with (16A). This factor, which gives the effect of recombination on the current, is plotted in Fig. 10. Recombination gives an increase in current flow, but the effect is small for the normal rate of surface recombination, which corresponds to $k = \lambda\rho \sim .035$.

APPENDIX B

Calculation of Hole Flow for Arbitrary Hole Concentration

In the text it was assumed that the concentration of holes was sufficiently small so that the first term in the brackets of Eq. (26) could be neglected in comparison with unity, yielding Eqs. (28) and (29). We give

here the general integration of Eqs. (26) and (27) for p arbitrarily large. Equation (26) may be written in the form:

$$i_p = -\text{grad}\,\psi, \tag{1B}$$

where

$$\psi = kT\mu_p\left[\frac{2bp}{b+1} - \frac{b(b-1)N_f}{(b+1)^2}\log\left(1 + \frac{(b+1)p}{bN_f}\right)\right]. \tag{2B}$$

Equation (27) then becomes:

$$\nabla^2\psi = 0. \tag{3B}$$

The radial solution of this equation corresponding to a total current I_p is:

$$\psi = \psi_\infty + I_p/2\pi r. \tag{4B}$$

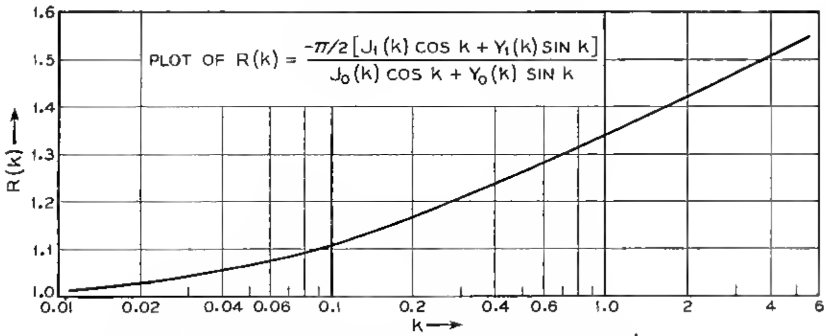

Fig. 10.—Correction factor for surface recombination.

The constants I_p and ψ_∞ are determined from the boundary conditions (30) and (31) of the text corresponding to $r = r_b$ and $r = \infty$. These conditions give:

$$\psi_\infty = kT\mu\left[\frac{2bp_i}{b+1} - \frac{2(b-1)N_f}{(b+1)^2}\log\left(1 + \frac{(b+1)p_i}{bN_f}\right)\right], \tag{5B}$$

$$I_p = 2\pi r_b\,(\psi(r_b) - \psi_\infty),$$

$$= 2\pi r_b\left[\frac{2b(p_b - p_i)}{b+1} - \frac{b(b-1)N_f}{(b+1)^2}\log\frac{bN_f + (b+1)p_b}{bN_f + (b+1)p_i}\right]. \tag{6B}$$

This equation is the appropriate generalization of Eq. (33) of the text. Since the equations are no longer linear, they do not apply strictly to the added hole concentration. However, if the normal hole concentration, p_0, is small, p_0 will be negligible in comparison with p_{ba} and p_a when the equa-

tions are not linear. Accordingly, to a close approximation, we may take for the added hole current:

$$I_{pa} = 2\pi r_b \left[\frac{2b(p_{ba} - p_a)}{b + 1} - \frac{b(b - 1)}{(b + 1)^2} \log \frac{bN_f + (b + 1)p_{ba}}{bN_f + (b + 1)p_a} \right], \tag{7B}$$

which is the generalization of Eq. (34) of the text.

The value of p_{ba} and thus of I_{pa} may then he found hy equating this expression with that of Eq. (3) for I_{pa}. This procedure yields the transcendental equation:

$$p_{ba} = -a \left[\frac{2b(p_{ba} - p_a)}{b + 1} - \frac{b(b - 1)N_f}{(b + 1)^2} \log \frac{bN_f + (b + 1)p_{ba}}{bN_f + (b + 1)p_a} \right], \tag{8B}$$

where a is again defined hy Eq. (6) of the text. This equation must be solved in general hy numerical methods for a particular case. The equation simplifies for p_a either large or small compared with N_f. The latter case is treated in the text. The opposite limiting case of large hole concentrations is treated below.

For p_a large compared with N_f, the logarithm may be neglected, so that

$$p_{ba} = -2ab(p_{ba} - p_a)/(b + 1). \tag{9B}$$

If, as in the text, it is assumed that a is small in comparison with unity, there results:

$$p_{ba} = 2abp_a/(b + 1), \tag{10B}$$

and, using (3):

$$I_{pa} = -[2b/(b + 1)]p_a kT\mu_p A/r_b . \tag{11B}$$

This differs from (7) by a factor $2b/(b + 1)$. The equation corresponding to (8) will have this additional factor, and also the expression for the conductance, G, which, for large hole concentrations is:

$$G = G_0 + [2b/(b + 1)](\alpha\beta\sigma_0 A/br_b)(p_a/n_0), \tag{12B}$$

in place of (38) of the text. Equation (16) which relates floating potential and conductance is general, and applies for arhitrary hole concentration.